Chapter (4) Factors and Polynomials

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1. The polynomial p(x) is $x^4 - 2x^3 - 3x^2 + 8x - 4$.

   (i) Show that p(x) can be written as (x - 1)$(x^3 - x^2 - 4x + 4)$. [1]

$(x-1)(x^3-x^2-4x+4)$

$= x^4-x^3-4x^2+4x-x^3+x^2+4x-4$

$= x^4-2x^3-3x^2+8x-4$ (shown)

   (ii) Hence write p(x) as a product of its linear factors, showing all your working. [4]

$p(x) = (x-1)(x^3-x^2-4x+4)$

$x = -2, 2, 1$

$f(x) = x^3-x^2-4x+4$

$f(1) = 1-1-4+4 = 0$

$(x-1)$ is a factor of $f(x)$

$$\begin{array}{r|l} & x^2 - 4 \\ \hline x-1 & x^3 - x^2 - 4x + 4 \\ & - \; x^3 + x^2 \\ \hline & 0 - 4x + 4 \\ & -4x + 4 \\ \hline & 0 \end{array}$$

$f(x) = (x-1)(x^2-4)$

$= (x-1)(x-2)(x+2)$

$p(x) = (x-1)(x-1)(x-2)(x+2)$

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2. It is given that p(x) = $2x^3 + ax^2 + 4x + b$, where a and b are constants. It is given also that $2x + 1$ is a factor of p(x) and that when p(x) is divided by $x - 1$ there is a remainder of -12.

(i) Find the value of a and of b. [5]

$p(-\frac{1}{2}) = 0$ $\quad$ $p(1) = -12$

$p(x) = 2x^3 + ax^2 + 4x + b$

$p(1) = 2 + a + 4 + b$

$-12 = 6 + a + b$

$a + b = -18$ — 1

$p(-\frac{1}{2}) = -\frac{2}{8} + \frac{a}{4} - \frac{4}{2} + b$

$0 = -\frac{1}{4} + \frac{a}{4} - 2 + b$ $\quad (\times 4)$

$0 = -1 + a - 8 + 4b$

$0 = a + 4b - 9$

$a + 4b = 9$ — 2

$\cancel{a} + b = -18$

$\cancel{a} + 4b = 9$

$-3b = -27$

$b = 9 \leftarrow$

$a + b = -18$

$a = -18 - 9$

$a = -27 \leftarrow$

(ii) Using your values of a and b, write p(x) in the form $(2x+1)$q(x), where q(x) is a quadratic expression. [2]

$p(x) = 2x^3 - 27x^2 + 4x + 9$

$$
\begin{array}{r|l}
 & x^2 - 14x + 9 \\
\hline
2x + 1 & 2x^3 - 27x^2 + 4x + 9 \\
 & \underline{-(2x^3 + x^2)} \\
 & -28x^2 + 4x \\
 & \underline{-(-28x^2 - 14x)} \\
 & 18x + 9 \\
 & 18x + 9
\end{array}
$$

$p(x) = (2x+1)(x^2 - 14x + 9)$

(iii) Hence find the exact solutions of the equation p(x) = 0. [2]

$p(x) = 0$

$2x + 1 = 0$

$x = -\frac{1}{2}$

$x^2 - 14x + 9 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{14 \pm \sqrt{160}}{2}$

$= \frac{14 + 4\sqrt{10}}{2} = 7 + 2\sqrt{10}$

$\frac{14 - 4\sqrt{10}}{2} = 7 - 2\sqrt{10}$

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3. A polynomial p(x) is $ax^3 + 8x^2 + bx + 5$, where *a* and *b* are integers. It is given that 2*x*-1 is a factor of p(x) and that a remainder of –25 is obtained when p(x) is divided by *x* + 2.

(i) Find the value of *a* and of *b*. [5]

$p(\tfrac{1}{2}) = 0$

$p(\tfrac{1}{2}) = \frac{a}{8} + \frac{8}{4} + \frac{b}{2} + 5$

$0 = \frac{a}{8} + 2 + \frac{b}{2} + 5$

$0 = \frac{a}{8} + \frac{b}{2} + 7$

$0 = a + 4b + 56$

$a + 4b = -56$ — ①

$p(-2) = -25$

$p(-2) = -8a + 32 - 2b + 5$

$-25 = -8a - 2b + 37$

$-62 = -8a - 2b$

$62 = 8a + 2b$ — ②

$16a + \cancel{4b} = 124$

$\ominus\ a \ominus \cancel{4b} = \oplus -56$

$15a = 180$

$a = 12$

$a + 4b = -56$

$12 + 4b = -56$

$4b = -68$

$b = -17$

(ii) Using your values of *a* and *b*, find the exact solutions of p(x) = 5. [2]

$p(x) = 12x^3 + 8x^2 - 17x + 5$

$12x^3 + 8x^2 - 17x + 5 = 5$

$12x^3 + 8x^2 - 17x = 0$

$x(12x^2 + 8x - 17) = 0$

$x = 0$ or $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-8 \pm \sqrt{64 + 816}}{24}$

$x = \frac{-2 + \sqrt{55}}{6}$ or

$x = \frac{-2 - \sqrt{55}}{6}$

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4. **Without using a calculator**, solve the equation $6c^3 - 7c^2 + 1 = 0$. [5]

$f(c) = 6c^3 - 7c^2 + 1$ $\qquad c = -\frac{1}{3}, 1, \frac{1}{2}$

$f(1) = 6 - 7 + 1$

$= 0$

$(c-1)$ is a factor of $f(c)$

$$\begin{array}{r|l} & 6c^2 - c - 1 \\ c-1 & \cancel{6c^3} - 7c^2 + 0c + 1 \\ & \cancel{6c^3} - 6c^2 \\ & \quad -\cancel{c^2} + 0c \\ & \quad -\cancel{c^2} + c \\ & \qquad -c + 1 \\ & \qquad -c + 1 \\ & \qquad\quad 0 \end{array}$$

$f(c) = (c-1)(6c^2 - c - 1)$ $\qquad 3c \quad + \quad 1 \quad 2c$

$0 = (c-1)(3c+1)(2c-1)$ $\qquad 2c \quad - \quad 1 \quad 3c$

$c = 1, -\frac{1}{3}, \frac{1}{2}$

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5. The cubic equation $x^3 + ax^2 + bx - 36 = 0$ has a repeated positive integer root.

(i) If the repeated root is $x = 3$ find the other positive root and the value of a and of b. [4]

$(x-3)(x-3)(x-c) = x^3 + ax^2 + bx - 36$

$-3 \times -3 \times -c = -36$

$-9c = -36$

$c = 4$ other root $= 4$

$(x-3)(x-3)(x-4)$

$= (x^2 - 6x + 9)(x-4)$

$= x^3 - 6x^2 + 9x - 4x^2 + 24x - 36$

$= x^3 - 10x^2 + 33x - 36$

$a = -10$, $b = 33$

(ii)There are other possible values of a and b for which the cubic equation has a repeated positive integer root. In each case state all three integer roots of the equation. [4]

$x = 3, 3, 4$

$6 \times 6 \times 1 = 36$

$1 \times 1 \times 36 = 36$

$2 \times 2 \times 9 = 36$

$x = 6, 6, 1$

$x = 1, 1, 36$

$x = 2, 2, 9$

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6. The remainder obtained when the polynomial p(x) = $x^3 + ax^2 - 3x + b$ is divided by x + 3 is twice the remainder obtained when p(x) is divided by $x - 2$. Given also that p(x) is divisible by x + 1, find the value of a and of b. [5]

$P(-1) = 0$

$-1 + a + 3 + b = 0$

$a + b + 2 = 0$

$a + b = -2$ — 2

$P(-3) = 2\,P(2)$

$P(-3) = -27 + 9a + 9 + b$

$P(2) = 8 + 4a - 6 + b$

$9a + b - 18 = (4a + b + 2) \times 2$

$9a + b - 18 = 8a + 2b + 4$

$a - \cancel{b} = 22$ — 1

$a + \cancel{b} = -2$

$2a = 20$

$a = 10$

$a + b = -2$

$b = -12$

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7. **Do not use a calculator in this question.**

It is given that $x+4$ is a factor of $p(x) = 2x^3 + 3x^2 + ax - 12$. When p(x) is divided by $x-1$ the remainder is b.

(i) Show that $a = -23$ and find the value of the constant b. [2]

$$P(-4) = -128 + 48 - 4a - 12$$
$$0 = -92 - 4a$$
$$4a = -92$$
$$a = -23 \text{ (shown)}$$

$$P(1) = 2 + 3 - 23 - 12$$
$$b = -30$$

(ii) Factorise p(x) completely and hence state all the solutions of p(x) = 0. [4]

$$\begin{array}{r|l} & 2x^2 - 5x - 3 \\ \hline x+4 & 2x^3 + 3x^2 - 23x - 12 \\ & -(2x^3 + 8x^2) \\ \hline & -5x^2 - 23x \\ & -5x^2 - 20x \\ \hline & -3x - 12 \\ & -3x - 12 \\ \hline & 0 \end{array}$$

$$p(x) = (x+4)(2x^2 - 5x - 3)$$
$$= (x+4)(x-3)(2x+1)$$

$$\begin{array}{ccc} 1 & -3 & -6 \\ 2 & 1 & +1 \end{array}$$

$$p(x) = 0$$

$x - 4 = 0$ or $x - 3 = 0$ or $2x + 1 = 0$

$x = 4$, $x = 3$, $x = -\frac{1}{2}$

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8. It is given that $x + 3$ is a factor of the polynomial $p(x) = 2x^3 + ax^2 - 24x + b$. The remainder when $p(x)$ is divided by $x - 2$ is -15. Find the remainder when $p(x)$ is divided by $x + 1$. [6]

$p(-3) = -54 + 9a + 72 + b$

$0 = 18 + 9a + b$

$9a + b = -18$ — 1

$p(2) = 16 + 4a - 48 + b$

$-15 = -32 + 4a + b$

$4a + \cancel{b} = 17$ — 2

$-\ 9a \overset{-}{+} \cancel{b} = \overset{+}{-}18$ — 1

$-5a = 35$

$a = -7$

$4a + b = 17$

$-28 + b = 17$

$b = 45$

$p(x) = 2x^3 - 7x^2 - 24x + 45$

$p(-1) = -2 - 7 + 24 + 45$

$= 60$

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9. $p(x) = 2x^3 + 5x^2 + 4x + a$

$q(x) = 4x^2 + 3ax + b$

Given that p(x) has a remainder of 2 when divided by 2x + 1 and that q(x) is divisible by x + 2 ,

(i) find the value of each of the constants *a* and *b*. [3]

$$p(-\tfrac{1}{2}) = \frac{-1}{4} + \frac{5}{4} - 2 + a$$

$$2 = 1 - 2 + a$$

$$a = 2 + 1$$

$$= 3$$

$$q(-2) = 16 - 3 \times 3 \times 2 + b$$

$$0 = 16 - 18 + b$$

$$0 = -2 + b$$

$$b = 2$$

Given that r(x) = p(x) - q(x) and using your values of *a* and *b*,

(ii) find the exact remainder when r(x) is divided by 3x - 2 . [3]

$$p(x) = 2x^3 + 5x^2 + 4x + 3$$

$$\ominus \quad q(x) = 4x^2 + 9x + 2$$

$$r(x) = 2x^3 + x^2 - 5x + 1$$

$$r(\tfrac{2}{3}) = 2 \times \frac{8}{27} + \frac{4}{9} - \frac{10}{3} + 1$$

$$= -\frac{35}{27}$$

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10. The polynomial p(x) = $ax^3 + 17x^2 + bx - 8$ is divisible by 2x-1 and has a remainder of -35 when divided by x + 3.

(i) By finding the value of each of the constants a and b, verify that a = b. [4]

$p(\frac{1}{2}) = \frac{a}{8} + \frac{17}{4} + \frac{b}{2} - 8$

$\times 8$

$0 = a + 34 + 4b - 64$

$a + 4b = 30$ — ①

$p(-3) = -27a + 153 - 3b - 8$

$-35 = 145 - 27a - 3b$

$-180 = -27a - 3b$

$-60 = -9a - b$

$-240 = -36a - 4\cancel{b}$

$30 = a + 4\cancel{b}$

$-210 = -35a$

$6 = a$

$a + 4b = 30$

$6 + 4b = 30$

$4b = 24$

$b = 6 = a$ (shown)

Using your values of a and b,

(ii) find p(x) in the form (2x-1)q(x), where q(x) is a quadratic expression, [2]

$$\begin{array}{r|l} & 3x^2 + 10x + 8 \\ \hline 2x-1 & 6x^3 + 17x^2 + 6x - 8 \\ & -(6x^3 - 3x^2) \\ \hline & 20x^2 + 6x \\ & -(20x^2 - 10x) \\ \hline & 16x - 8 \\ & 16x - 8 \\ \hline & 0 \end{array}$$

$p(x) = (2x-1)\underbrace{(3x^2+10x+8)}_{q(x)}$

$3 \quad 4 + 4$

$1 \quad 2 + 6$

(iii) factorise p(x) completely, [1]

$p(x) = (2x-1)(3x^2 + 10x + 8)$

$= (2x-1)(3x+4)(x+2)$

(iv) solve $a\,sin^3\theta + 17\,sin^2\theta + b\,sin\theta - 8 = 0$ for $0 < \theta < 180$. [3]

$(2\sin\theta - 1)(3\sin\theta + 4)(\sin\theta + 2) = 0$

$\sin\theta = \frac{1}{2}$ $\quad$ $\sin\theta = \frac{-4}{3}$ (Reject) $\quad$ $\sin\theta = -2$ (reject)

$\theta = \sin^{-1}(1/2)$

$= 30, 180 - 30^\circ$

$= 30^\circ, 150^\circ$